ANALYSIS OF ENERGY-TRANSPORT PERFORMANCE OF MACHINES VIA THE RESISTANCE CONCEPT

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(Received 5 August 1963)

Abstract—Mollier diagrams such as the P-h (pressure-enthalpy) and the h-s (enthalpy-entropy) graphically indicate energy relationships under fluid-flow circumstances and greatly enhance the understanding of the processes. They particularly depict energy allocations to work and to resultant energy-wealth differences for machines involving flow under pressure-difference conditions. With associated heat-exchanger equipment, the "energy-transport" through machines subsequently may well be clarified via an amplified resistance-concept, with energy-flow responsive to a specified motivating-force or "potential-difference". This approach then lends itself well to the prediction of operating performance in energy-flow complexes particularly under off-design conditions, and suggests modes of treatment via rheo-electric simulation techniques. Two examples of energy-transport for machines coupled to heat-exchanger are discussed, namely refrigeration and steam-turbine power generation.

NOMENCLATURE

Refrigeration

q, hourly heat transfer, heat flow;

- $\Delta t_{t,c}$, overall temperature-difference between condensing vapor and inlet coolant;
- R_t , overall heat-transfer resistance for condenser;
- R_c , heat-exchange resistance corresponding to water-equivalent E_c ;
- W, hourly mass coolant flow-rate;

c, specific heat of coolant fluid;

 E_c , Wc, "water equivalent" for coolant fluid;

- U, overall coefficient of heat transfer for refrigerant vapor vs. coolant;
- A, heat-exchanger surface area;
- R_{UA} , heat-transfer resistance through surface area;
- *R*, thermal resistance, $\Delta t/q$.

Power generation

- q_f , hourly total enthalpy energy-transfer, Btu/h, for condenser-heater;
- Δh_t , overall enthalpy-difference for condenser-heater;
- Δh_T , total enthalpy-difference;
- W_c , rate of process-fluid flow lb/h;
- *u*, overall rate of energy-transfer based

on Δh , Btu/(h/ft² Δh_m); q_e , total exhaust-energy, Btu/h.

WHATEVER the formal name for the internal plus pressure-volume energy composite-term h = u + Pv/J, known in the past as "heat-content" or "total-heat" and latterly as "enthalpy", the energy concept and its practical applications so richly pioneered by Richard Mollier has been a real corner-stone of engineering-technology development. The use of Mollier diagrams such as the *P-h* (pressure-enthalpy) and the *h-s* (enthalpy-entropy) has greatly contributed to the visualization and understanding of energyflow processes in machines, as found in systems for refrigeration and for power-production.

This paper is humbly dedicated to the memory of the famous technologist whose contributions to thermodynamics made its understanding via his diagrams so realistic—Richard Mollier.

The general solution of thermal-energy flow processes, particularly under off-design conditions, is fraught with many complications; under these circumstances, recourse to analysis via the resistance-concept of energy-flow offers some distinct advantages. Study of complexes involving refrigerating machines becomes increasingly redeeming when undertaken via the resistance-concept; the same is the case for power-production systems. It is the special purpose of this paper to illustrate this aspect through extensions of the concept, hitherto associated primarily with heat conduction, to include energy-flow machines and heatexchangers, via energy-transport resistance and heat-exchange resistance respectively. Through the extended resistance-concept, complicated processes may be simulated analytically with electrical flow and its attendant resistance circuitry, that is, by electrical analogy particularly with the development of resistive computer circuits in mind.

These further concepts of energy-flow resistance thus have become additional tools for rheo-electric analysis of integrated energy-flow systems, and will be illustrated through two examples: (a) refrigeration, involving vaporcompression via a compressor discharging into a vapor-condenser; and (b) power-production via a turbine discharging into a condenser-heater. The first case is developed in terms of temperature-difference representing the motivating force; and the second, in terms of enthalpydifference; the treatment will be general, since details may be further explored in some of the specified citations [7, 10].

In the field of resistance-concept analysis, especially by rheo-electric applications, many studies have been carried out in heat-conduction problems; reference may be made to different steady-state developments in this realm by the author, using both geometrical sheet-analog [1, 2, 3] and network [4] methods. Further extension has covered the heat-exchanger problem [5], combined heat and vapor flow [6], the energy-transport principle applied to a refrigerating plant [7, 8] as herein discussed, analysis of a process-fluid pipe network flow-distribution system [9], and then analysis of a heat-and-power process-plant complex [10] as also explored in this present paper. Transient problems have also been studied in a variety of problems, several of which may be noted [11, 12].

ENERGY-FLOW IN A REFRIGERATION COMPLEX

The vapor-compression refrigeration system depicted in Fig. 1 consists of an evaporator



at low temperature in which the required refrigeration effect is produced. The refrigerant vapor flows to a vapor-compressor with mechanical power input, whereby the vapor-pressure is elevated to such a value that it will liquefy under heat-exchange operations; a pressure-drop "expansion-valve" reduces the refrigerant pressure so as to permit evaporation at the lower temperature. Typical pressure-temperature (P-t)relations are shown in Fig. 2, and in Fig. 3 in t-P arrangement for clarity. The overall idealized temperature patterns are shown in Fig. 4. In Fig. 5 is depicted the all-important Mollier P-h diagram, showing the energy relationships for the refrigerant in going from the lower







FIG. 6.

evaporating temperature to the higher condensing temperature via the compressor which functions as the energy-transport device. Under fixed evaporator and ideal-cycle conditions, the pertinent cycle enthalpy-difference values for varying condenser pressure are shown typically in Fig. 6. It is to be noted that $\Delta h_{\rm cond} = \Delta h_{\rm evan} +$ Δh_{work} ; thus the total condenser energy is made up of transported evaporator-energy augmented by the necessary compressor work.

The total energy-flow rate transported to the condenser for its heat-exchange absorption depends on the real performance-characteristics of the compressor in terms of the quantity of refrigerant gas actually compressed per hour in

conjunction with the pressure-difference between the evaporator and the condenser. For a pistondisplacement compressor this includes the effect of the real volumetric efficiency E_v , [7], variable with pressure-difference. With fixed suction (evaporator) pressure conditions, the hourly quantity of refrigerant pumped depends on the condensing pressure in addition to the general dimensional characteristics of the machine along with the volumetric efficiency; see Fig. 7. Coupled with the cycle characteristics as represented in Fig. 6, the hourly compressor energies appear typically as in Fig. 8, as a function of temperature.

The function of the condenser in a refrigeration

circuit is to absorb the energy from the compressor-discharged refrigerant at the condensing temperature. The condensing temperature represents the result of equilibrium balance such that the energy-flow-rate absorbed in the condenser just equals that contributed by the compressor system. Thus the equilibrium condensing temperature is sensitive, on one hand, to the condenser heat-flow performance characteristics (involving heat-transfer surface area A, and hourly overall heat-transfer coefficient U as a function of coolant flow-rate and temperature, etc.), and on the other hand, to the energy-transport performance-characteristic of the compressor. This temperature is thus common to both equipments in each case. This relationship may well be visualized ultimately in the rheoelectric simulation circuit.

Condenser heat-exchange resistance

In the condenser, such as of the shell-and-tube type, the coolant, entering at its supply temperature and flowing through at a given rate in sensible heat-exchange, suffers a corresponding temperature rise. The heat-flow takes place through the separating walls between the refrigerant and the coolant, and depends in total on the extent of their temperature difference and the heattransfer characteristics of the equipment.

Disregarding the effect on condenser performance of superheat (compensated for in the



$$q = \Delta t_{t,c} / R_t \tag{1}$$

$$R_t = R_c / [1 - \exp(-R_c / R_{UA})]$$
 (2)

$$R_c = 1/(Wc) = 1/E_c$$
 (3)

$$R_{UA} = 1/(UA) \tag{4}$$

$$\Delta R_t = R_t - R_c \tag{5}$$

where, in addition to previously identified symbols, W = hourly mass coolant flow-rate, c is the fluid specific heat (E_c , "water-equivalent", = Wc). These relationships are shown on a general basis in Fig. 10, with values plotted against coolant flow-rate W, for normal temperature conditions. R_{UA} represents the overall heat-transfer resistance and is variable with W, R_c is the equivalent "heat-exchange" resistance, and R_t is the total energy-flow resistance so important in the circuit studies and procedures.

Compressor energy-transport resistance

Considering the above energy relationships









for the compressor, just as the resistance-concept has been extended to cover the total performance of a heat-exchanger versus temperature-difference $\Delta t_{t,c}$, so may it similarly be extended to cover energy-transport in a machine, as for example, in a compressor. Equivalent resistance values may be used to represent the performance. Assuming a fixed evaporator pressure and corresponding temperature, and a variation of compressor performance, in terms of condensing temperature in lieu of condensing pressure, the typical Fig. 8 has been evolved. As a next step, using the corresponding temperature-difference Δt as based on the fixed evaporator temperature,



the transformed information of Fig. 8 is shown in Fig. 11. Resistance values may then be shown for the compressor performance by dividing the compressor Δt by the hourly heat quantities:

$$R = \Delta t/q. \tag{6}$$

In Fig. 11 such typical resistance values are shown against temperature-difference, as follows:

- R_{evap} = resistance value corresponding to hourly evaporator heat-flow;
- R_{cond} = resistance value corresponding to hourly condenser energy;
- R_{work} = resistance value corresponding to hourly compressor work.

Resistance circuits are very useful for understanding of energy-flow processes; this includes both thermal-resistance and rheo-electric circuits. For the refrigeration energy-transport phase via compressor to condenser, there is, as noted, transport of q_{evap} plus q_{work} energy through the R_{cond} up to the condensing-temperature level. From here the energy flows down to the inletcoolant level through condenser-resistance R_t via $\Delta t_{t,c}$, with the energy-flow q in the condenser equal to q_{cond} of the compressing unit. Fig. 12 as a temperature-resistance diagram illustrates these relationships, the strength of the energyflow rate being illustrated by the straight line over the $\Delta t - R$ range: $q = \Delta t / R$ and $\Delta t = qR$. It thus follows that for the condenser and its q, $\Delta t_{t,c} = (\Delta t/R)(R_t)$, as illustrated in the small key diagram at the base of the *t*-R chart, and then reproduced in the upper part for completeness. Thus $\Delta t_{t,c}$ shows as stemming from the condensing temperature to produce the corresponding and necessary heat-flow q for the condenser down to the inlet-coolant temperature. Δt_c then shows the temperature-rise of the coolant, under its flow-rate W, in the condenser.

ENERGY-FLOW IN A POWER AND PROCESS-HEAT COMPLEX

Power generation and process-fluid heating are often combined into a single complex in process-plant designs. A steam turbine may exhaust directly to a vapor-condensing fluidheater, to serve, in effect, as a power-producing "reducing-valve". Here the performance of the turbine in steam consumption will be directly



FIG. 12.



FIG. 13.

influenced by the resultant back-pressure which is the operating pressure on the heater; conversely, the heater performance under its operating conditions of fluid-flow rate and initial temperature must be directly dependent on the associated performance of the steam turbine in series with it.

In the prediction of the overall performancecharacteristics of such an integrated complex, the resistance-concept is again of special value. In heat-flow and processes like the previous case, temperature-difference has been used as the motivating force; in the present powerprocess the motivating force selected to implement the resistance-concept is the thermodynamic property of "enthalpy-difference suggested by the Mollier h-s diagram for energyflow values [10].

The equipment arrangement is shown in Fig. 13. Herein the steam turbine, coupled to its electric generator, and drawing vapor from a steam-generator, exhausts directly to a vaporcondensing fluid-heater of the shell- and tubetype. With fluid flow at a given rate and inlet temperature, typically process water, the fluid will be heated to some resultant temperature. If a particular specified final temperature be required for the fluid, it may be passed into a second heater using direct high-pressure steam through a temperature-controlled reducing valve, as shown in the figure. The primary interest here is in the steam circuit for the turbine and its connected first-stage heater (condenser).

Condenser-heater resistance

In a perfect fluid-heater, the outlet fluid temperature would be that of the condensingvapor source; thus, in working with enthalpy potential difference Δh , the fluid enthalpy h_f corresponding to the vapor-temperature may be utilized as the source level of energy flow. Fig. 14 shows the conventional heat-exchanger temperature-pattern with its counterpart values in terms of enthalpy. This approach permits utilization of enthalpy-difference as motivating force for the flow of system energy.

The hourly energy-transfer q_t for the condenser-heater may be set up [10] via an exchanger resistance R_t , similarly as was done for the previous example, except that it is developed on



an enthalpy base:

$$q_t = \Delta h_t / R_t \tag{7}$$

$$R_t = R_c/[1 - \exp\left(-\frac{R_c}{R_{uA}}\right)] \quad (8)$$

$$R_c = 1/W_c \tag{9}$$

$$\Delta R_t = R_t - R_c \tag{10}$$

where W_c is the rate of process-fluid flow in lb/h; $R_{uA} = 1/(uA)$ with $u = Btu/(h/ft^2 \Delta h_m)$ and variable with the fluid-flow rate through the condenser-heater tubes, and A = transfer area in ft².

Turbine energy-transport resistance

For a power-plant cycle, the value of the exhaust pressure has a significant effect on the efficiency of transformation of the available Δh_T into work Δh_w , and thus on the magnitude of the exhaust energy Δh_e . This is illustrated in the *h*-s Mollier diagram Fig. 15. With decreasing values of back pressure the work-energy value Δh_w increases. Distinction must be made between ideal-cycle versus real-cycle performance (with internal losses involving efficiency η). It is also to be noted that the realized work per unitpound of cycle fluid leads to the significant rate-of-steam-consumption value, that is, to the familiar "water-rate". This is the hourly steam flow-rate per kilowatt-hour power production. Thus:

water rate,
$$WR = 3415/(\eta \Delta h_{w, \text{ ideal}}) \text{ lb/kWh.}$$
(11)



For a turbine consuming a total amount of steam, S lb/h, under a given back pressure condition the exhaust-energy per pound would be $\Delta h_{e, \text{ real}}$, and the total exhaust-energy per hour would be q_e

$$q_e = S(\Delta h_{e, \text{ real}}). \tag{12}$$

The exhaust-energy-transport resistance R_e , associated with Δh_T as defining the "potential-difference" here, may now be defined:

$$R_e = \Delta h_T / q_e \tag{13}$$

where q_e , represents the total hourly exhaustenergy discharged from the turbine, and, for a given installation, this is dependent on Δh_T which is subject to the effect of the back-pressure liquid enthalpy value h_f .

Energy-flow circuit

A rheo-electric simulation circuit for the power-process-heat complex is shown in Fig. 16. In particular it shows R_e as a variable resistance in the turbine-condenser-heater section, in series with the heat-exchanger resistance R_t defined in equation (8). As in the previous example,



electric resistance values here are in scale relationship to the respective thermal resistance values. (In the absence of a standard designation for the unit of energy-flow resistance, for convenience here and as suggested by electric practice, the term "enthohm" is used; "thohm" is similarly used with heat-flow.)

For given circumstances, the value of workenergy and of exhaust-energy are each dependent on the overall value of Δh_T . Hence, in the resistive electrical circuit, inasmuch as R_e depends on the resultant value of Δh_T established under circuit conditions, it must be adjusted accordingly. The circuit thus shows an automatic servo-controlled resistance value R_{e} , its characteristic depending on the particular turbine involved in the study. It must be specifically pointed out that whatever exhaust-energy as such flows through the turbine must also "flow" through the heat-exchanger down to the level of the incoming process-fluid at Point 1 in the diagram. It is also to be noted that the circuit is truly an energy-flow circuit, precisely as the name indicates.

With "ground" representing 0 value of the enthalpy scale, and an appropriate d.c. voltage applied to represent the initial steam enthalpy h_g for Point 7, the initial fluid-enthalpy $h_{c,1}$ (Point 1) may be established either by manual control of the adjusting resistance, or through servo-control via voltage E_1 . Under balance conditions, E_3 represents the intermediate condensing-temperature enthalpy h_f , and E_2 the



voltage representative of $h_{c,2}$ of the outlet process-fluid.

As noted in the subsequently cited illustrative example, since the associated value of kilowatt output is constant, with the only variation for the turbine-plant taking place in the exhaustenergy condenser-heater section of the circuit, this latter section is the one of particular interest. To re-iterate: the energy-flow in the power circuit is regarded as constant, and may be so set, with an automatic current regulator to maintain it so, if desired. Although it is not of primary interest here, the electrical circuit diagram also shows the additional provisions for the further heating of the process-fluid in a second-stage heater to some desired value. This uses a heater-circuit similar to that of the primary heater. The required outlet potential, via E_{11} , serves to govern the necessary current-flow through a variable resistance that simulates a pressure-reducing valve, with the second-stage inlet fluid constantly regulated to coincide with E_2 , either manually or automatically.

Whereas in this heat-power analogue circuit the adjustment of the different controlling resistances to maintain values of h and of R_e as a function of Δh_T may be accomplished manually, still this is a matter of manual dexterity, greatly simplified, as noted, by the use of automatic equipment.

Illustrative example

The citation of an illustrative example employed in an earlier presentation [10] will serve to clarify the operations further.

A system such as represented by Fig. 13 operates with steam at 250 psi and 200°F superheat ($h_g = 1319$ Btu/lb) and employs a base-load turbine designed for a continuous output of 1000 kW under the varying backpressure conditions that might be realized. Internal turbine efficiency, $\eta = 0.65$, may be considered as constant over the range of backpressure variation. The condenser-heater has 2250 ft² of surface. Plant performance-predictions as to condensing temperature t_F and fluidoutlet temperature $t_{c,2}$ are desired at the processfluid flow rate of $W_c = 335\,000$ lb (water) per hour and for varying inlet-water temperatures $t_{c,1}$ from 40° to 130°F. Fig. 17 shows the performance-characteristics of the turbine under the fixed admission conditions specified, i.e. $h_g = 1319$ Btu/lb, for both ideal and real ($\eta = 0.65$) cases. The steam consumption values for 1 kW output are specified, namely "water-rate" in pounds of steam per kWh, along with the corresponding exhaust-energy Δh_e Btu/lb, and plotted against Δh_T . In effect, this represents performance under different back-pressure conditions.

Fig. 18 shows the operating characteristics of the condenser-heater, and illustrates their variation with the coolant fluid-flow rate W_c . Significant here is that the value of u, the energy-transfer surface rate, varies with the W_c flow rate, as illustrated in the $(uA)_c$ curve; R_c , as per equation (9), obviously depends on the flow rate; R_t via equation (8), is likewise involved. Thus the values of R_t and R_c for 335 000 lb/h are directly available.

Fig. 19 is developed directly from the specification of 1000 kW turbine output and from the values given in Fig. 17; it translates the turbine performance values for ultimate use in the analysis. It shows, for the real case, the variation of exhaust-energy discharged from the 1000-kW output turbine to the condenser-heater, q_e Btu/h, along with the corresponding value of exhaust-energy transport resistance R_e plotted against Δh_T .

Fig. 20 shows the result predictions for the condenser-heater outlet water temperature $t_{c,2}$ and the condensing temperature t_F , as plotted against the varying inlet-water temperature $t_{c,1}$.

ACKOWLEDGEMENT

For his help in the preparation of this paper, grateful acknowledgement is made to the author's associate, John A. McCague. This paper reports the results of another phase of the investigations supported in part by a grant-in-aid from The Refrigeration Research Foundation.

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Résumé—Les diagrammes de Mollier tels que ceux donnant *P-h* (pression enthalpie) et *h-s* (enthalpieentropie) fournissent graphiquement les relations énergétiques dans des conditions d'écoulement fluide déterminées et contribuent grandement à la compréhension des phénomènes. Ils déterminent en particulier l'énergie qu'il faut fournir et les différences d'énergie utilisables dans le cas d'une machine comportant un fluide s'écoulant entre deux sources à pressions différentes. En y associant un échangeur thermique le transport d'énergie dans les machines peut être éclairci à l'aide d'une notion généralisée de résistance, qui fait correspondre le flux d'énergie à une différence de potentiel.

Cette façon de voir se prête elle-même alors bien à la prévision des performances des systèmes thermodynamiques, particulièrement à ceux fonctionnant en circuit ouvert, et suggère des modes d'études par analogies rhéo-électriques. Deux exemples de transport d'énergie par des machines couplées à des échangeurs sont étudiés: la réfrigération et la production de puissance par machine à vapeur.

Zusammenfassung—Mollierdiagramme wie das P-h (Druck-Enthalpie) und das h-s (Enthalpie-Entropie) Diagramm zeigen grafisch die Energiezusammenhänge bei strömenden Medien und vertiefen das Verständnis der dabei auftretenden Vorgänge. Insbesondere geben sie Energieaufteilungen in Arbeit und resultierende Energieinhaltsdifferenzen für druckabbauende Strömungsmaschinen an. Mit einem angeschlossenen Wärmeübertrager lässt sich der "Energietransport" durch Maschinen mit Hilfe eines erweiterten Widerstandskonzepts gut erklären, wobei der Energiefluss von einer bestimmten Treibkraft oder "Potentialdifferenz" abhängt. Diese Näherung eignet sich gut zur Bestimmung des Betriebsverhaltens von Energieflusskomplexen, besonders unter abnormalen Bedingungen und legt eine Behandlung mit Hilfe rheoelektrischer Simulationsverfahren nahe. Zwei Beispiele des Energietransports werden für Maschinen mit angeschlossenen Wärmeübertragern diskutiert, nämlich Kühlung und Damp'krafterzeugung.

Аннотация—Диаграммы Молье, такие как P - h (давление-энтальпия) uh - s (энтальпияэнтропия), дают графическое представление об энергетических соотношениях в условиях течения жидкости и эначительно способствуют пониманию происходящих процессов. При подключении теплообменника «перенос энергии» через агрстаты можно определить при помощи упрощенного понятия сопротивления, когда на поток энергии влияет определенная движущая сила или «разность потенциалов». Этот метод удобен для определения режима работы в системах с потоком энергии, особенно при нерасчётных условиях, и позволяет рассматривать явления с помощью реоэлектрического моделирования. Рассмотрены два примера переноса энергии для агрегатов, присоединенных к теплообменникам, а именно : производство холода и генерация энергии в паровой

турбине.